A Stochastic Model of the Earth's Interior

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Abstract

Spectral analysis of observational data by Pelletier (2002) indicates that the geomagnetic field has a variance spectrum which follows a 1/f power law and that a return period of geomagnetic reversals similar to that observed is a direct consequence of such a power law spectrum. A stochastic model of the earth's interior is proposed in which the number of assumptions is kept to a minimum, i.e. specific heat, thermal conductivity, radiogenic heating and density are constant. Despite this simplicity, complex behaviour occurs as a consequence of further assumptions: that Rayleigh-Bernard convection cells form spontaneously and at random in the outer liquid core as heat builds up from radioactive sources, that each of these cells spontaneously generates its own magnetic field by a MHD dynamo effect, that each cell melts the solid mantle immediately above it because of the extra heat being convected outward from the hot core and that in this way each convection cell propagates upwards through the otherwise solid mantle at a speed determined by the solution of the Stefan problem for a liquid-solid boundary. The upward-moving, liquid-in-solid convection cells formed in this way are proposed as the primary mechanism by which the core is cooled. The totality of convection cell MHD dynamo fields is proposed as the origin of the geomagnetic field which will have a 1/f spectrum and experience reversals similar to those observed. Because cooling is a stochastic process, there will be times when the earth is heating faster than it is cooling and vice versa. Hence there will be times when the volume and surface of the earth are expanding and new crust is formed and there will be other times when the surface is contracting and the crust, being too large for the smaller surface, is forced to ramp up, wrinkle and subduct in order to be accommodated by the smaller area.

The Spectrum of the Geomagnetic Field

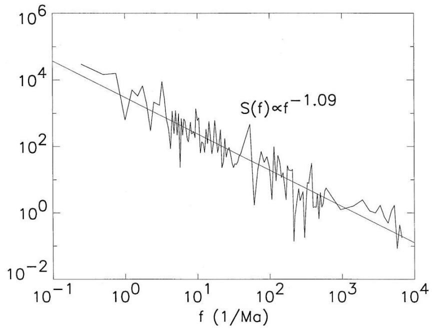

Figure 1 The variance density spectrum of the intensity of the Earth's magnetic field. (Fig 8 of Pelletier, 2002)

Figure 1 shows that:

- The power spectrum of the intensity of the Earth's magnetic field is well approximated by a 1/f dependence implying that the driving mechanism is stochastic rather than deterministic.
- This suggests that turbulent convection is involved.
- Geomagnetic field reversals observed in the time domain (Fig. 2) are a direct consequence of this 1/f spectral distribution.
- A successful physical theory must account for this spectral form.
- Geothermal processes are not steady-state.

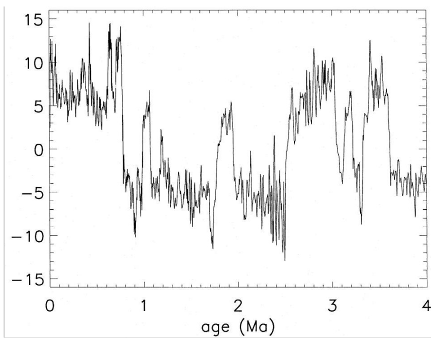

Figure 2 Time series of the intensity of Earth's magnetic field as inferred from sediment cores from the past 4 Ma showing geomagnetic field reversals (Fig 7 of Pelletier, 2002).

A Conduction-Only Model

A one-dimensional heat transport model was developed by integrating the heat equation in which there was no convective transport, only conduction. Uniform radioactive heating ($H_r = 2.10^{-8} W m^{-3}$) and uniform conductivity ($\kappa = 1.0 W.m^{-1}.K^{-1}$) were assumed. These are commonly accepted values (Gerya, 2010).

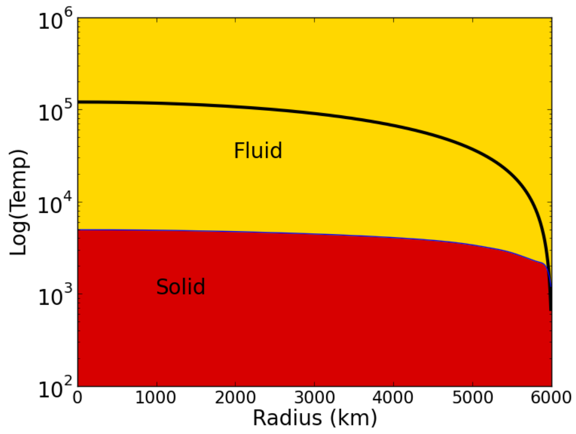

Figure 3 The temperature of the interior of the Earth in the absence of convection.

The resulting temperature distribution is shown in Figure 3. Note that:

- Core temperature $= 100,000\,^{\circ}$C.
- The temperature curve intersects the solidus at 160 km depth.
- Almost the entire interior would be fluid and as hot as a star.
- In the present epoch the liquid core radius is observed to be only 3500 km, hence convection must play a major role in cooling the interior of the earth.

Liquid-in-solid Convection

Conventionally the mantle is assumed to be plastic enough to allow convection, but sufficiently rigid to support seismic S-waves. We propose an alternative - Liquid-in-Solid Convection - as depicted in idealized form in Figure 4.

Note that:

- The outer core experiences Rayleigh-Bernard convection (R-B convection cells form when a liquid is in a vertical temperature gradient).
- Liquid-in-solid convection (LISC) cells are unlikely to be the neat ellipsoids shown in Figure 4 but but irregular blobs which may well be lopsided or even meander like a river.
- Rayleigh number calculations indicate that LISC cells can range from 100s of meters to 1000s of kilometers in linear dimension.
- Larger cells will generate their own magnetic field as self-exciting magnetohydrodynamic dynamos.

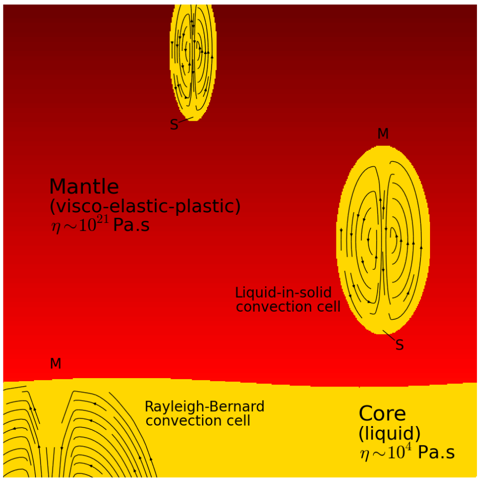

Figure 4 A conceptualization of liquid-in-solid convection cells formed in the lower mantle from Rayleigh-Bernard convection cells in the core. M: regions of melting due to increasing temperature. S: regions of solidification due to cooling and increasing pressure. Note that this diagram is highly idealized and does NOT represent numerical model output.

It is impossible to predict when or where a new cell will start up nor how big it will be because turbulent convection is a stochastic process. All physical quantities associated with LISC will also be stochastic and have pink or red spectra. These include ascent speed, volume, magnetic field strength, and temperature.

The Implications of a Stochastic Model

- The geomagnetic field is the sum of the individual fields of all the LISC and RB convection cells in the core and mantle convection cell. It must also be a stochastic quantity which varies over time in a random manner.
- The total volume is the sum of the volumes of the core, the solid mantle and all the LISC cells in the mantle.It must also be a stochastic quantity which varies over time in a random manner according to the proportion of molten material in the core and mantle.
- Hence sometimes the earth is expanding and at other times it is contracting. When the earth is expanding new crust forms at mid-ocean ridges (Fig. 5) but when it is contracting it is forced to wrinkle and subduct.
- Mountain building occurs when the earth contracts.
- Sea floor spreading occurs when the earth expands - recent magnetic stripes average 6km across indicating a total expansion of 36km in the earth's circumference, i.e. roughly 0.1 percent variation in circumference during each expansion phase.
- Glatzmeier and Roberts(1995) account for isolated field reversals but not the spectrum seen in Figure 1. Their deterministic model could be upgraded to include stochastic forcing as described here.

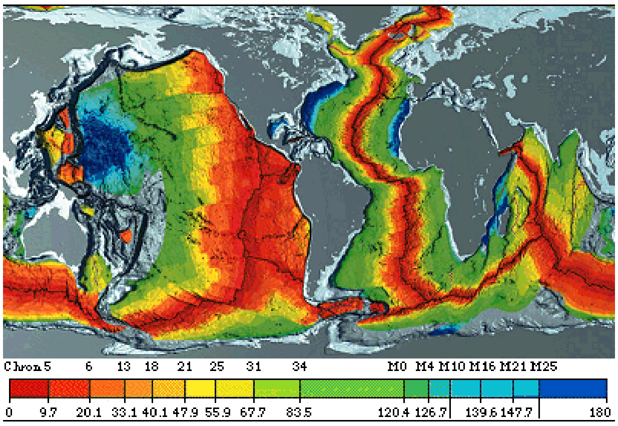

Figure 5 Map of the age of the seafloor based on the magnetic field recorded in the oceanic crust during its formation at mid-oceanic ridges. Red indicates a young seafloor, whereas blue is used for the oldest oceanic crust. (Source: National Geophysical Data Center)

Experimental Verification

- The most sensitive measure of the earth's volume is Length of Day which has varied little over the last couple of centuries but is known to have varied in the past.
- The reality of liquid-in-solid convection is contentious but its existence or otherwise can be verified in laboratory experiments, e.g. by subjecting a large block of wax to a vertical temperature gradient.
- Numerical modeling is deterministic and so cannot fully emulate the stochastic aspects of convection. However modeling of individual LISC cells under various viscosity and solidus scenarios could provide important insights.

References

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